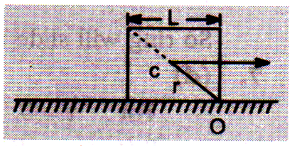
WEEKLY TEST RANKER'S BATCH TEST - 04 RAJPUR
SOLUTION Date 22-09-2019

[PHYSICS]

1. The block would start rotating about an axis passing through the point O



Since no external torque acts on the block, its angular momentum is conserved.

Angular momentum of the block before hitting the ridge $= mv(a/2)$

Angular momentum of the block after hitting the ridge $= I_0\omega$

The moment of inertia, I_0, of the block about the axis passing through the point O is

$$I_O = I_C + Mr^2$$

$$= \frac{Ma^2}{6} + M\left(\frac{a^2}{4} + \frac{a^2}{4}\right) = \frac{Ma^2}{6} + \frac{Ma^2}{2}$$

$$= \frac{2}{3}Ma^2$$

2. $$\vec{v}_{PQ} = \vec{v}_P - \vec{v}_Q = \left(\frac{8\sqrt{3}}{2}\hat{i} - \frac{8}{2}\hat{j}\right) - \left(\frac{6\sqrt{3}}{2}\hat{i} + \frac{6}{2}\hat{j}\right)$$

$$= \sqrt{3}\hat{i} - \sqrt{7}\hat{j}$$

$$\vec{L} = \vec{r} \times \vec{P} = (-10\,\hat{i}) \times m(\sqrt{3}\,\hat{i} - 7\hat{j}) = 70m\hat{k}$$

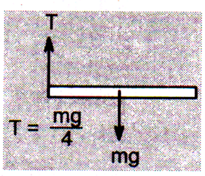
$$m10^2\omega = m \times 70 \Rightarrow \omega = 0.7 \text{ rad/s}$$

3.

$$Mg - T = MA$$

$$Mg\frac{L}{2} = \frac{ML^2}{3}\alpha$$

$$\Rightarrow \quad \alpha L = \frac{3g}{2}$$

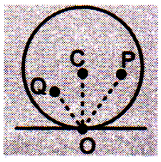


Also $\quad a = \alpha\frac{L}{2}$

Solving we get

$$T = \frac{Mg}{4}$$

4. In case of pure rolling bottom most point is the instantaneous axis of pure rotation about which the whole body rotates with same angular velocity. So farther the point from O larger will be the velocity.

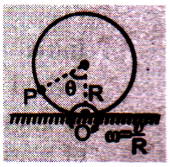


5. This can be assumed as a pure rotation about point of contact say O with angular velocity $\omega = \frac{v}{R}$, where R is the radius of hoop.

Speed of P will be: $v_P = (OP)\omega = \left(2R\sin\frac{\theta}{2}\right)\omega$

or $v_P = (2R\omega)\sin\left(\frac{\theta}{2}\right)$

or $v_P = 2v\sin\left(\frac{\theta}{2}\right)$



6. Since velocity and its magnitude depends upon reference frame so P and KE should depend upon reference frame.

7. When boy will walk, friction force will act forward, boys shifts right and the block does not move (since lower surface is rough). Therefore, centre of mass shifts.

8. To change the linear momentum external force is required.

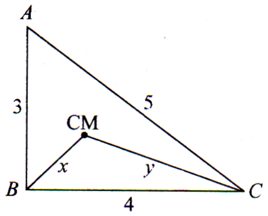
9. $$v = \frac{J}{2m} \Rightarrow \omega = \frac{Jl}{2ml^2} = \frac{J}{2ml}$$

$$\therefore \quad v_A = v + l\omega = \frac{J}{2m} + \frac{J}{2m} = \frac{J}{m}$$

10. In sphere P, the point of contact has tendency to move towards left w.r.t. surface and hence, friction acts on it towards right.
In spheres Q and R, the point of contact has tendency to move towards right w.r.t. surface and hence, friction acts on both of them towards left.
In sphere S, the point of contact has tendency to move towards left due to rotation and towards right due to translatory motion. It has not been specified in question whether v is less than or greater than ωR. Hence, friction on it may act towards left or right.

11. Moment of inertia is more when mass is farther from the axis. In case of axis BC, mass distribution is closest to it and in case of axis AB mass distribution is farthest. Hence

$$I_{BC} < I_{AC} < I_{AB} \Rightarrow I_P > I_B > I_H$$



$$I_C = I_{CM} + my^2 = I'_B - mx^2 + my^2$$

Here I'_B is moment of inertia of the plate about an axis perpendicular to it and passing through B.

$\Rightarrow \quad I_C = I'B + m(y^2 - x^2) = I_P + I_R + m(y^2 - x^2)$

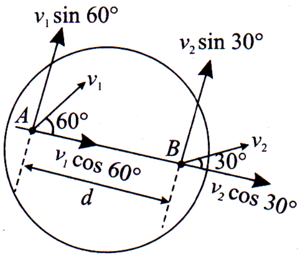
It means $I_C > I_P + I_B$ also $I_C > I_P$

$\therefore \quad I_C > I_P > I_B > I_H$

12. For rigid body, separation between the two points remains the same.

$$v_1 \cos 60° = v_2 \cos 30°$$

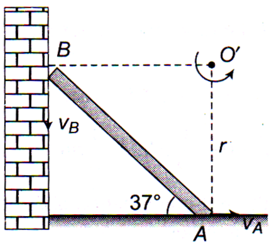
$$\frac{v_1}{2} = \frac{\sqrt{3}\,v_2}{2} \Rightarrow v_1 = \sqrt{3}\,v_2$$



$$\omega_{disc} = \left|\frac{v_2 \sin 30° - v_1 \sin 60°}{d}\right| = \left|\frac{\frac{v_2}{2} - \frac{\sqrt{3}v_1}{2}}{d}\right|$$

$$= \left|\frac{v_2 - \sqrt{3} \times \sqrt{3}v_2}{2d}\right| = \frac{2v_2}{2d} = \frac{v_2}{d} \Rightarrow \omega_{disc} = \frac{v_2}{d}$$

13.



Velocity of end A, $\vec{v}_A$ is horizontal, while velocity of the end B, $\vec{v}_B$ is vertical downward. For finding the position of instantaneous centre of rotation. Drop perpendiculars to the directions of $\vec{v}_A$ and $\vec{v}_B$ at points A and B respectively. The intersection point I be the instantaneous centre.

$$x = \ell \cos 37° = 5 \times \frac{4}{5} = 4 \text{ m}$$

$$y = \ell \sin 37° = 5 \times \frac{3}{5} = 3 \text{ m}$$

Hence, coordinate of I.C.R. is (4 m, 3 m).

14. Torque about the CM is caused by friction because the lever arm of the weight force is zero:

$$\tau = fR = I\alpha$$

$$f = \mu n = \mu mg \cos\theta$$

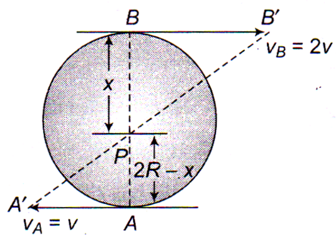
$$\mu = \frac{f}{mg\cos\theta} = \frac{I\alpha/R}{mg\cos\theta}$$

$$= \frac{\left(\frac{2}{3}g\sin\theta\right)\left(\frac{1}{2}mR^2\right)}{R^2 mg\cos\theta} = \left(\frac{1}{2}\tan\theta\right)$$

15. Since, there is no relative sliding between the cylinder and the planks 1 and 2, the points A and B of the disc will move with velocities equal to the velocities of the respective surfaces.

$\vec{v}_A = -v\hat{i}$ and $\vec{v}_B = -2v\hat{i}$ respectively. Joining A and B and A' and B' we find the point P as IAR. Then we have the similar triangles PAA' and PBB'. Using the properties of similar triangles we have

$$\omega = \frac{BB'}{AA'} = \frac{BP}{AP}$$



where $AA' = v_A = v$, $BB' = v_B = 2v$, $AP = 2R - x$ and $BP = x$.

Then, we have $\frac{2v}{v} = \frac{x}{2R-x}$

This gives, $x = \frac{4R}{3}$

Hence, IAR is located at a distance of 4R/3 below the top of the disc.

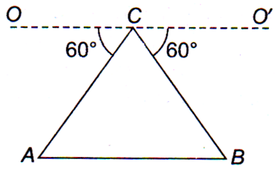
As we know $\omega = \frac{v_{AB}}{AB}$

$$v_{AB} = |\vec{v}_A - \vec{v}_B| = |-v\hat{i} - 2v\hat{i}| = 3v \text{ and } AB = 2R$$

This gives $\omega = \frac{3v}{2R}$ clockwise.

16 Moment of inertia of each rod AC and BC about the given axis OO' is

$$I_{AC} = I_{BC} = \frac{m\ell^2}{3}\sin^2 60° = \frac{m\ell^2}{4}$$



and M.I. of rod AB about the given axis OO' is

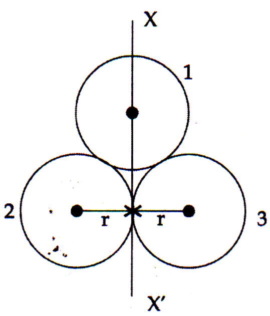
$$I_{AB} = m\left(\frac{\ell\sqrt{3}}{2}\right)^2 = \frac{3}{4}m\ell^2$$

Hence, $I = I_{AC} + I_{BC} + I_{AB} = \frac{m\ell^2}{4} + \frac{m\ell^2}{4} + \frac{3}{4}m\ell^2 = \frac{5}{4}m\ell^2$

17. If we take boat and both persons as a system, there is no external force acting on the system. The center of mass of the system is initially at rest and will be at rest as there is no external force acting on it to displace center of mass. Hence there is no shifting of center of mass.

18. Moment of inertia of circular disc = $\frac{1}{2}mR^2$. Thus, as the distance between the centre and the point increases, the moment of inertia increases.

19



$I_{xx'} = I_1 + I_2 + I_3$

$$\frac{2}{3}mr^2 + \left(\frac{2}{3}mr^2 + mr^2\right) + \left(\frac{2}{3}mr^2 + mr^2\right)$$

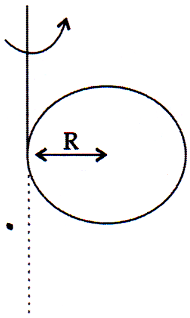
(Using parallel axis theorem)

$\Rightarrow I_{xx'} = 2mr^2 + 2mr^2 = 4mr^2$

20. For conservation of angular momentum about origin

$\sum \vec{\tau}_{net} = 0 \Rightarrow \vec{r}\vec{F} = 0 \Rightarrow \alpha = -1$

21.

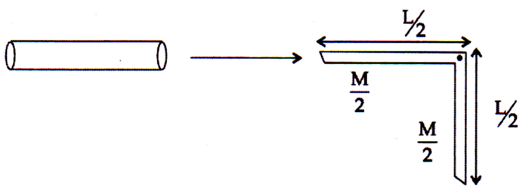


According to parallel axis theorem,

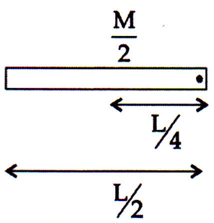
$$I = \overbrace{\frac{MR^2}{2}}^{\text{about centre}} + MR^2$$

$$= \frac{3}{2}MR^2$$

22.



Moment of Inertia of one part

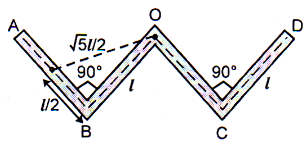


$$= \frac{ML^2}{3}$$

Here, $m = \frac{m}{2}$

$L = L/2$

23. The given structure can be broken into 4 parts



For AB : $I = I_{CM} + m \times d^2 = \frac{m\ell^2}{12} + \frac{5m}{4}\ell^2$; $I_{AB} = \frac{4}{3}ml^2$

For BO : $I = \frac{m\ell^2}{3}$

∴ For composite frame : (by symmetry)

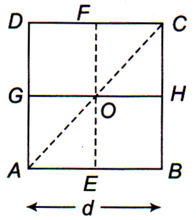
$I = 2[I_{AB} + I_{OB}] = 2\left[\frac{4m\ell^2}{3} + \frac{m\ell^2}{3}\right] = \frac{10}{3}m\ell^2$.]

24. Let the each side of square lamina is d.

So, $I_{EF} = I_{GH}$ (due to symmetry)

and $I_{AC} = I_{BD}$ (due to symmetry)

Now, according to theorem of perpendicular axis,



$$I_{AC} + I_{BD} = I_0$$

$\Rightarrow$ $2I_{AC} = I_0$ ...(i)

and $I_{EF} + I_{GH} = I_0$

$\Rightarrow$ $2I_{EF} = I_0$ ...(ii)

From Eqs. (i) and (ii), we get $I_{AC} = I_{EF}$

$$\therefore \quad I_{AD} = I_{EF} + \frac{md^2}{4}$$

$$= \frac{md^2}{12} + \frac{md^2}{4} \quad \left(\text{as } I_{EF} = \frac{md^2}{12}\right)$$

So, $I_{AD} = \dfrac{md^2}{3} = 4I_{EF}$

25. As the disc is in combined rotation and translation, each point has a tangential velocity and a linear velocity in the forward direction.

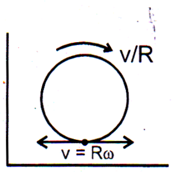
From figure

v_{net} (for lowest point) = v – Rω = v – v = 0.

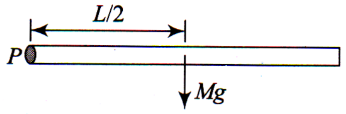
and Acceleration = $\dfrac{v^2}{R} + 0 = \dfrac{v^2}{R}$

(Since linear speed is constant)

Hence (D).



26 Taking torque about P



$$Mg\,\frac{L}{2} = \left(\frac{ML^2}{3}\right)\alpha$$

Hence $\alpha = \dfrac{3g}{2L}$

27. Apply law of conservation of angular momentum.

$$I_1\omega_1 = I_2\omega_2$$

In the given case $I_1 = MR^2$

and $I_2 = MR^2 = 2mR^2$

also $\omega_1 = \omega$

Then $\omega_2 = \frac{I_1}{I_1}\omega = \frac{M}{M+2m}\omega$

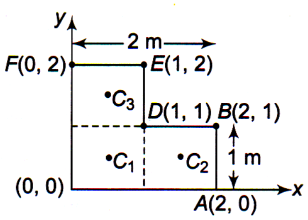
28. The theorem of parallel axis for moment of inertia.

$$I = I_{CM} + Mh^2$$

$$I = I_0 + M\left(\frac{L}{2}\right)^2$$

$$= I_0 + \frac{ML^2}{4}$$

29. Choosing the x and y axes as shown in the figure. The coordinates of the vertices of the L-shaped lamina is as shown in the figure.



Divide the L-shaped lamina into three squares each of side 1 m and mass 1 kg (∵ the lamina is uniform). By symmetry, the centres of mass C_1, C_2 and C_3 of the squares are their geometric centres and have coordinates $C_1\left(\frac{1}{2},\frac{1}{2}\right)$, $C_2\left(\frac{3}{2},\frac{1}{2}\right)$ and $C_3\left(\frac{1}{2},\frac{3}{2}\right)$ respectively.

The coordinates of the centre of mass of the L-shaped lamina is

$$X_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{1\times\frac{1}{2} + 1\times\frac{3}{2} + 1\times\frac{1}{2}}{1+1+1} = \frac{5}{6}\text{m}$$

$$Y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$= \frac{1\times\frac{1}{2} + 1\times\frac{1}{2} + 1\times\frac{3}{2}}{1+1+1} = \frac{5}{6}\text{m}$$

30. $L = mr^2\omega$, Now Given $r' = \frac{r}{2}$ and $\omega' = \omega$

$$L' = m\omega\frac{r^2}{4} = \frac{L}{4}$$

31. As said, $(KE)_{rot}$ remains same.

i.e., $\frac{1}{2}I_1\omega_1^2 = \frac{1}{2}I_2\omega_2^2$

$\Rightarrow \quad \frac{1}{2I_1}(I_1\omega_1)^2 = \frac{1}{2I_2}(I_2\omega_2)^2$

$\Rightarrow \quad \frac{L_1^2}{I_1} = \frac{L_2^2}{I_2}$

$\Rightarrow \quad \frac{L_1}{L_2} = \sqrt{\frac{I_1}{I_2}}$

but $\quad I_1 = I, I_2 = 2I$

$\therefore \quad \frac{L_1}{L_2} = \sqrt{\frac{I}{2I}} = \frac{1}{2}$

or $\quad L_1 : L_2 = 1 : \sqrt{2}$

32. The moment of inertia about an axis passing through centre of mass of disc and perpendicular to its plane is

$$I_{CM} = \frac{1}{2}MR^2$$

Where M is the mass of disc and R its radius. According to theorem of parallel axis, MI of circular disc about an axis touching the disc at its diameter and normal to the disc is

$$I = I_{CM} + MR^2$$

$$= \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

33. Using $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ (ii)

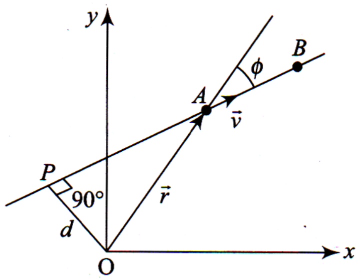
Given, $\alpha = 3.0$ rad/s^2, $\omega_0 = 2.0$ rad/s, $t = 2$s

Hence, $\theta = 2 \times 2 + \frac{1}{2} \times 3 \times (2)^2$

or $\quad \theta = 4 + 6 = 10$ rad

34 From the definition of angular momentum,

$$\vec{L} = \vec{r} \times \vec{p} = rmv \sin\phi\,(-\vec{k})$$



Therefore, the magnitude of L is

$$L = mvr \sin\phi = mvd$$

where $d = r\sin\phi$ is the distance of closest approach of the particle to the origin. As d is the same for both the cases, hence $L_A = L_B$.

35. The radius of gyration is given by

$$K = \sqrt{\frac{I}{M}}$$

For given problem $\frac{K_{\text{disc}}}{K_{\text{ring}}} = \sqrt{\frac{I_{\text{disc}}}{I_{\text{ring}}}}$ (i)

But I_{disc} (about its axis) $= \frac{1}{2}MR^2$

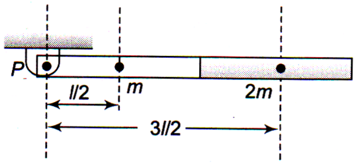
and I_{ring} (about its axis) $= MR^2$

where R is the radius of both bodies.

Therefore, Eq. (i) becomes

$$\frac{K_{\text{disc}}}{K_{\text{ring}}} = \sqrt{\frac{\frac{1}{2}MR^2}{MR^2}} = 1:\sqrt{2}$$

36. Moment of inertia of composite rod about P



$$I_P = \frac{m\ell^2}{3} + \frac{(2\text{m})\ell^2}{12} + 2m\left(\frac{3}{2}\ell\right)^2 = 5m\ell^2$$

Total kinetic energy k total $= \frac{1}{2} I_P \omega^2$

$$= \frac{1}{2}(5\text{m}\,\ell^2)\omega^2 = \frac{5}{2}m\ell^2\omega^2 = 250\text{ J}$$

37. Moment of inertia of rod of mass M and length l about its axis passing through one of its ends and perpendicular to it is

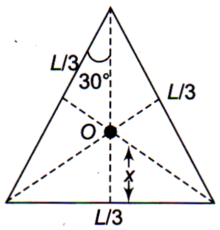
$$I = \frac{1}{3}Ml^2$$

As $I = Mk^2$ where k is the radius of the gyration

$$\therefore \quad Mk^2 = \frac{1}{3}Ml^2 \text{ or } k = \frac{l}{\sqrt{3}}$$

38. Mass of each side $= \frac{M}{3}$

$$x = \frac{1}{3}\left[\frac{L}{3}\cos 30°\right] = \frac{L/3}{2\sqrt{3}}$$



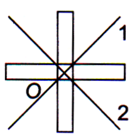
moment of inertia of one side about O:

$$I = \left[\frac{1}{12}\frac{M}{3}\left(\frac{L}{3}\right)^2 + \frac{M}{3}x^2\right] = \frac{ML^2}{162}$$

moment of inertia of whole triangle about $O : 3I = \frac{ML^2}{54}$

39. We have to find I_1?

$$I_1 = I_2 \quad (1)$$

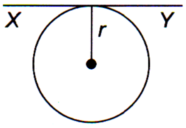


$$I_0 = \frac{ML^2}{12} + \frac{ML^2}{12} = \frac{ML^2}{6}$$

now $I_1 + I_2 = I_0$ (2)

From (1) and (2), $I_1 = \frac{I_0}{2} = \frac{ML^2}{12}$

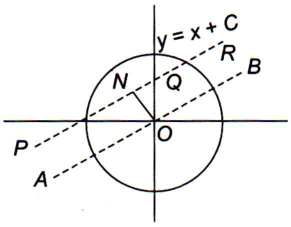
40. $L = 2\pi r \Rightarrow r = \frac{L}{2\pi}, m = \rho L$



$I_{xy} = \frac{3}{2}mr^2$

$= \frac{3}{2}\rho L\left(\frac{L}{2\pi}\right)^2 = \frac{3}{8}\frac{\rho L^3}{\pi^2}$

41. $I_{PQR} = I_{AOB} + M\cdot(ON)^2 I_{PQR} = \frac{1}{4}MR^2 + M\cdot\left(\frac{c}{\sqrt{2}}\right)^2$



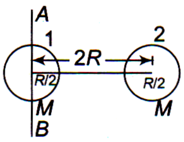
But $I_{PQR} = \frac{1}{2}MR^2 \therefore c = \pm\frac{R}{\sqrt{2}}$

Hence (b) is correct.

42. $I_{AB} = I_{1AB} + I_{2AB}$

$= \frac{2}{5}M\left(\frac{R}{2}\right)^2 + \frac{2}{5}M\left(\frac{R}{2}\right)^2 + M(2R)^2$

$= \frac{21}{5}MR^2$



43 Here, $v_0 = 420$ rpm $= 7$ rps

$\therefore \quad \omega_0 = 2\pi v_0 = 2\times\frac{22}{7}\times 7 = 44\ \text{rad s}^{-1}$

$\omega = 0, \alpha = -2\ \text{rad s}^{-2}$

$t = \frac{\omega - \omega_0}{\alpha} = \frac{-44}{-2} = 22\,\text{s}$

44 $\vec{a}_o = a\hat{i}, \vec{a}_{p/o} = \alpha r\hat{i} + \omega^2 r(-\hat{j})$

$\vec{a}_p = \vec{a}_o + \vec{a}_{p/o} = (a + \alpha r)\hat{i} - \omega^2 r\hat{j}$

$a_p = \sqrt{(a + \alpha r)^2 + (\omega^2 r)^2}$

45. At the highest point of the trajectory,

$$x = \frac{1}{2}R = \frac{v_i^2 \sin 2\theta}{2g} \text{ and } y = h_{\max} = \frac{(v_i \sin\theta)^2}{2g}$$

The angular momentum is then

$$\vec{L}_1 = \vec{r}_1 \times m\vec{v}_1$$

$$= \left[\frac{v_i^2 \sin 2\theta}{2g}\hat{i} + \frac{(v_i \sin\theta)^2}{2g}\hat{j}\right] \times mv_{xi}\hat{i}$$

$$= \frac{-mv_i^3 \sin\theta^2 \cos\theta}{2g}\hat{k}$$